Unit 3 College Algebra

1

1. The table below is a complete representation of relation $\mathcal{R}$. Is $\mathcal{R}$ one-to-one? Circle your answer.

$\mathcal{R}$	INPUT	-4	1	0	1	4
	OUTPUT	-2	-1	0	1	2

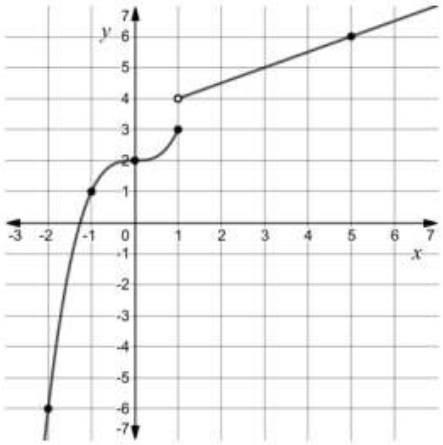
Yes (No)

We can find one element in the input that is associated with more than one element in the output. Specifically, the element 1 in the input is associated with -1 and 1 in the output.

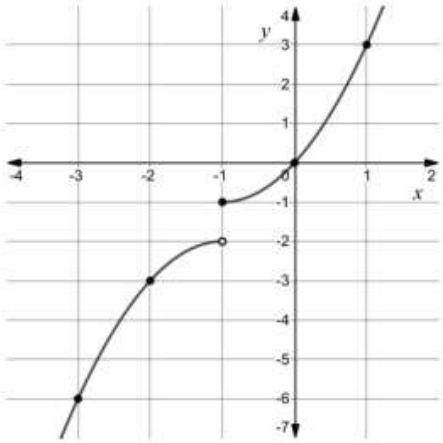
2

2. Use the graphs of the functions $f(x)$ and $g(x)$ to complete (a) – (d). If any of the values are not defined, mark them "UNDEF."

$y = f(x)$



$y = g(x)$



(a) $(f+g)(0) =$ **2**	(c) $(f \circ g)(-1) =$ **1**
(b) $\left(\frac{f}{g}\right)(-2) =$ **2**	(d) $g^{-1}(-3) =$ **-2**

Part-a)

$f(0) = 2$ and $g(0) = 0$ so $(f+g)(0) = 2 + 0 =$ **2**

Part-b)

$f(-2) = -6$ and $g(-2) = -3$ so $\left(\frac{f}{g}\right)(-2) = \frac{-6}{-3} =$ **2**

Part-c)

$g(-1) = -1$ so $(f \circ g)(-1) = f(-1) =$ **1**

Part-d)

$g(-2) = -3$ so $g^{-1}(-3) = -2$

3

3. Suppose $50,000 is invested at an interest rate of 5.1% per year, compounded monthly.

(a) Write the formula used to determine future value.

Part-a

$$F(t) = 50{,}000\left(1 + \frac{0.051}{12}\right)^{12t}$$

where $t =$ time in years and $F(t) =$ future value after t years

Part-b

(b) Find the future value after 10 years. Round to the nearest cent.

$$F(t) = 50{,}000\left(1 + \frac{0.051}{12}\right)^{120} = \$83{,}174.63$$

Part-c

(c) Derive the formula used to determine doubling time (*i. e.*, solve for t in Part 3a).

We want to express the equation in the form

$F(t) = 50{,}000(2)^{t/k}$ where k is the doubling time

We have

$$\left(1 + \frac{0.051}{12}\right)^{12t} = 2^{\log_2\left(1+\frac{0.051}{12}\right)^{12t}}$$

$$= 2^{12t\log_2\left(1+\frac{0.051}{12}\right)}$$

$$= 2^{12t\log\left(1+\frac{0.051}{12}\right)/\log 2}$$

$$= 2^{t/\left(\frac{\log 2}{12\log\left(1+\frac{0.051}{12}\right)}\right)}$$

Hence,

$$F(t) = 50{,}000(2)^{t/k}$$

where $k = \frac{\log 2}{12\log\left(1+\frac{0.051}{12}\right)} \approx 13.62$ years (doubling time)

Part-d

(d) Determine how long it will take to double the investment. Round to 2 decimal places.

We want to find t so that $F(t) = 100{,}000$

$$50{,}000\left(1+\frac{0.051}{12}\right)^{12t} = 100{,}000$$

Divide both sides by 50,000

$$\left(1+\frac{0.051}{12}\right)^{12t} = 2$$

Get the logarithm of both sides and apply property of logarithm

$$12t\left[\log\left(1+\frac{0.051}{12}\right)\right] = \log 2$$

$$t = \frac{\log 2}{12\left[\log\left(1+\frac{0.051}{12}\right)\right]}$$

$t \approx$ **13.62 years**

4

4. The Center for Disease Control (CDC) and World Health Organization (WHO) track cases of illnesses around the world as potential new pandemics are identified. The following fictitious data represents the number of reported cases of H1N1 during the first outbreak in 2009, where t is the time measured in days since the first reported incident and $P(t)$ is the number of reported cases.

t	2	4	6	8	10	12	14
$P(t)$	84	135	203	327	609	1014	1800

(a) Circle the appropriate type of regression for the data set above. Be sure to justify your answer.

linear exponential power

Justification:

Part-a

According to the calculator, the R^2 values for the linear, exponential, and power model are 0.8209, 0.998, and 0.8981, respectively. On the basis of the R2 value, we chose the **exponential regression**.

Part-b

(b) Use your graphing calculator to perform the regression from Part 4a. Write down the equation. Round to 2 decimal places.

According to the calculator, the best exponential regression equation that fits the data is

$$P(t) = 46.94(1.29)^t$$

Part-c

(c) Using the model developed in Part 4b, determine when the number of reported cases is 1,000,000. Round to the nearest day.

We want to solve for t so that $P(t) = 1{,}000{,}000$

$$46.94(1.29)^t = 1{,}000{,}000$$

Divide both sides by 46.94

$(1.29)^t = 1{,}000{,}000/46.94$

Get the logarithm of both sides and apply the property of logarithm

$t \log(1.29) = \log(1{,}000{,}000/46.94)$

$t = \frac{\log(1{,}000{,}000/46.94)}{\log(1.29)} \approx$ **39 days**

5

5. Follow the instructions below to find the inverse functions.

(a) Find a formula for $f^{-1}(x)$ given $f(x) = \frac{1}{3}x - 5$. (Use the 4-step procedure for full credit.)

Part-a

Let $y = f(x)$

$y = \frac{1}{3}x - 5$

Interchange x and y

$x = \frac{1}{3}y - 5$

Solve for y

Multiply both sides by 3

$3x = y - 15$

Add 15 to both sides

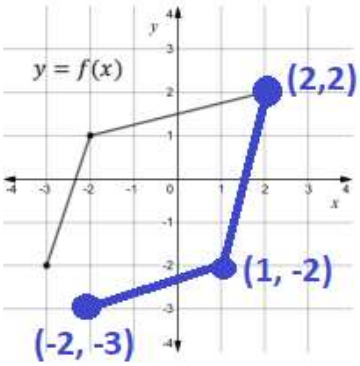
$3x + 15 = y$

$y = 3x + 15$

The inverse is $f^{-1}(x) = 3x + 15$

Part-b

(b) Use the graph of $y = f(x)$ to sketch a graph of $y = f^{-1}(x)$.

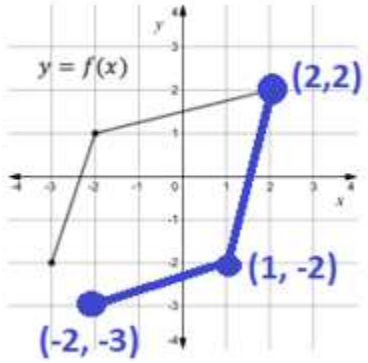


Explanation:

The points (-3, -2). (-2, 1), and (2, 2) are on the graph of f(x) and these are the endpoints of the two line segments.

We interchange the x and y-coordinates to get the corresponding points on the inverse function.

Hence, we plot the points (-2, -3), (1, -2), and (2, 2) then connect them by line segments



6

6. Use properties of logarithms to fully expand the following expression.

$$\log\sqrt{\frac{y+1}{x^3z^2}}$$

$$\log\sqrt{\frac{y+1}{x^3z^2}} = \log\left(\frac{y+1}{x^3z^2}\right)^{1/2}$$

$$= \frac{1}{2}\log\left(\frac{y+1}{x^3z^2}\right)$$
$$= \frac{1}{2}[\log(y+1) - \log x^3 - \log z^2]$$
$$= \frac{1}{2}[\log(y+1) - 3\log x - 2\log z]$$
$$= \frac{1}{2}\log(y+1) - \frac{3}{2}\log x - \log z$$

7

7. For the following, consider the functions:

$$f(x) = \frac{1}{x^2-9} \text{ and } g(x) = \sqrt{x+1}$$

(a) Find $h(x) = (f \circ g)(x)$.

Part-a

$$(f \circ g)(x) = f\left(\sqrt{x+1}\right)$$
$$= \frac{1}{\left(\sqrt{x+1}\right)^2 - 9}$$
$$= \frac{1}{x+1-9}$$
$$= \frac{1}{x-8}$$

Part-b

The domain of $g(x) = \sqrt{x+1}$ is $x \geq -1$ so that the expression inside the square root is non-negative.

x=8 makes the denominator of $\frac{1}{x-8}$ zero so we exclude 8 from the domain of $(f \circ g)(x)$

Answer: $[-1, 8) \cup (8, \infty)$

8

8. The barometric air pressure in inches of mercury (inHg) at a distance of x miles (mi) from the eye of a hurricane is modeled by the logarithmic function:

$$f(x) = 0.48 \ln(x + 1) + 27.$$

At what distance from the eye of the hurricane is the air pressure 27.65 inHg? Round to the nearest tenth.

We want to solve for x so that f(x) = 27.65

$0.48 \ln(x + 1) + 27 = 27.65$

Subtract 27 from each side

$0.48 \ln(x + 1) = 0.65$

Divide both sides by 0.48

$\ln(x + 1) = 0.65/0.48$

Express in exponential equation

$e^{0.65/0.48} = x + 1$

Subtract 1 from each side

$e^{0.65/0.48} - 1 = x$

$x = e^{0.65/0.48} - 1 \approx$ **2.9 miles**

9

9. City Engineers build an artificial lake designed to sustain up to 2000 fish. Upon completion, they release fish into the lake but fail to record the initial amount. The table below reflects fish counts beginning one month after the initial release.

Time (months)	t	1	3	6	9	12
Fish Population	P(t)	128	246	584	1098	1465

(a) Use a graphing calculator to fit a logistic function to the data. Round to 2 decimal places.

Part-a

According to the calculator, the best logistic function that best fits the data is

$$P(t) = \frac{1742.22}{1 + 19.87\, e^{-0.39\, t}}$$

Part-b

(b) Use the function to estimate the initial population (t = 0) and predict the population after 2 years (t = 24 months).

$$P(0) = \frac{1742.22}{1+19.87\ e^{0}} \approx 83$$

$$P(24) = \frac{1742.22}{1+19.87\ e^{-0.39\ (24)}} \approx 1739$$

The estimated initial population using the logistic model is 83 and the predicted population after 2 years is 1739.

Part-c

(c) According to your model, will the fish population ever reach 2000?

According to the model, the fish population will never reach 2000 because the horizontal asymptote is y = 1742.22 and the function according to the graph is increasing.

10

10. Ahmed stopped by the local bakery and bought 3 apple fritters, 2 berry scones and 1 cheese danish for $14.60. Maria visited the same bakery and was charged $16.90 for 2 apple fritters, 1 berry scone and 4 cheese danishes. Abigail paid $19.80 for 1 apple fritter, 5 berry scones and 3 cheese danishes.

(a) Convert the information above into a system of 3 equations.

Part-a

Let x = price of an apple fritter

y = price of a berry scone

z = price of a cheese danish

The system of 3 equations is

$$\begin{cases} 3x + 2y + z = 14.60 \\ 2x + y + 4z = 16.90 \\ x + 5y + 3z = 19.80 \end{cases}$$

Part-b

(b) Write your system as an augmented matrix.

We just copy the coefficients in the equation from the previous part to get the augmented matrix

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 14.60 \\ 2 & 1 & 4 & 16.90 \\ 1 & 5 & 3 & 19.80 \end{array}\right]$$

Part-c

(c) Use your graphing calculator to find the reduced row echelon form (RREF) of your matrix.

According to the calculator, the RREF is

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2.75 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2.35 \end{array}\right]$$

Part-d

(d) What are the solutions of your system? Interpret your result.

According to the RREF, the solution is (x, y, z) = (2.75, 2, 2.35)

This means that cost per order of apple fritter, berry scone, and cheese Danish are $2.75, $2, and $2.35, respectively.